the Robust Network Loading Problem with Dynamic Routing

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the problem

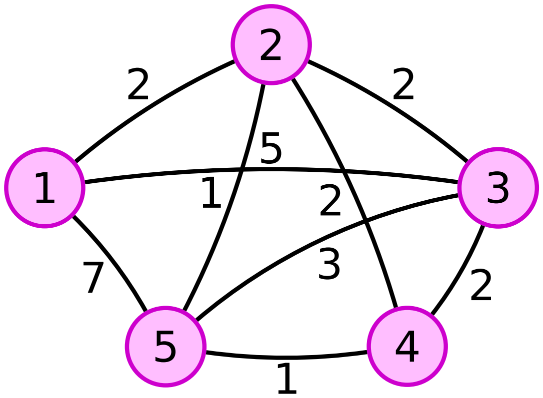
- description
- mathematical model

the branch-and-cut algorithm

- separation routines
- branch-and-cut heuristic

conclusions

- preliminary computational results
- future work



$d_{12} = 3.2$
$d_{15} = 5$
$d_{24} = 0.7$
$d_{34} = 1.1$
$d_{35} = 5.3$
$d_{45} = 5.3$
$d_{ij} = 0$ otherwise

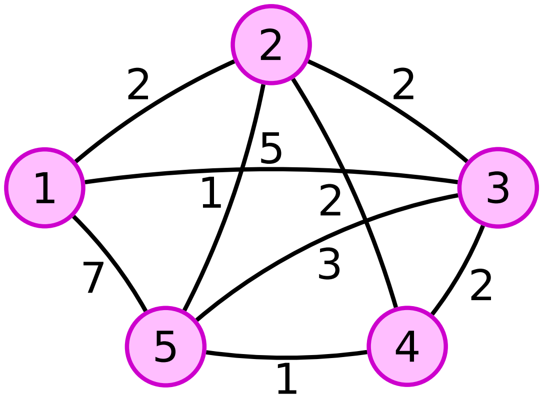
given

- a graph $G(V, E)$ with per-unit edge costs $c : E \to \mathbb{R}_+$
- a traffic matrix: set of point-to-point traffic demands (commodities)

problem

compute minimum cost integer capacities such that all the demands can be routed simultaneously on the network

the Robust Network Loading Problem



$$d_{12} = 3.2 \quad d_{15} = 5 \quad d_{24} = 0.7 \quad d_{34} = 1.1 \quad d_{35} = 5.3 \quad d_{45} = 5.3 \quad d_{ij} = 0$$

$$\ldots$$

$$d_{13} = 0.7 \quad d_{14} = 2.1 \quad d_{23} = 0.7 \quad d_{24} = 1.9 \quad d_{34} = 1.8 \quad d_{35} = 2 \quad d_{ij} = 0$$

given

- a graph $G(V, E)$ with per-unit edge costs $c : E \to \mathbb{R}_+$
- a set of traffic matrices D to be served non simultaneously

problem

compute minimum cost integer capacities such that every $d \in D$ can be supported

the demand set D

- D explicitly given (list of matrices)
- D implicitly described (polyhedral representation [1])

the hose polyhedron

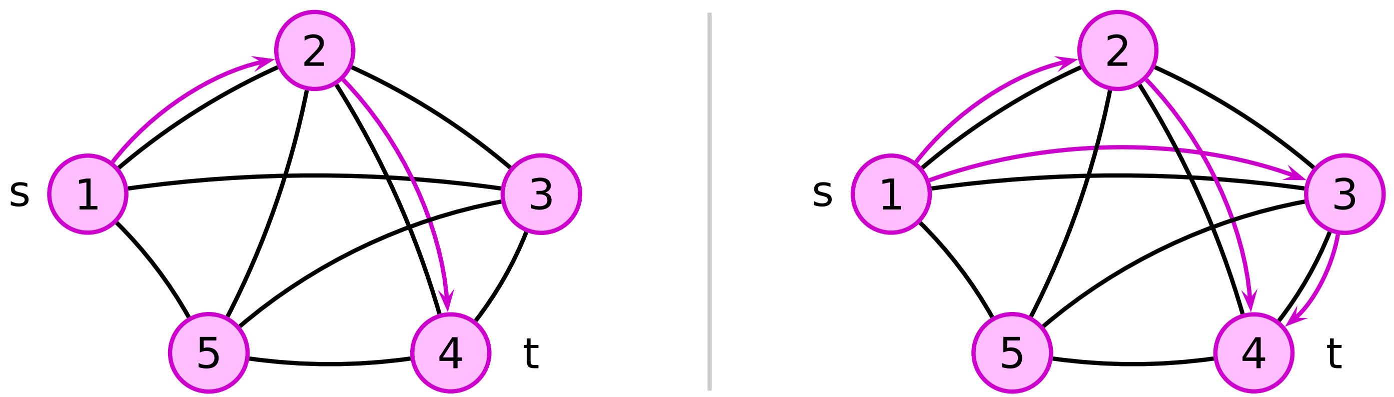
feasible demands must respect bounds on node traffic [2][3]

- symmetric: a single bound on the sum of the incoming and outgoing traffic
- asymmetric: two bounds, one for the incoming traffic and one for the outgoing traffic

[1] Ben-Ameur, Kerivin (2005)
[2] Duffield, Goyal, Greenberg, Mishra, Ramakrishnan van der Merwe (1999)
[3] Fingerhut, Suri, Turner (1997)

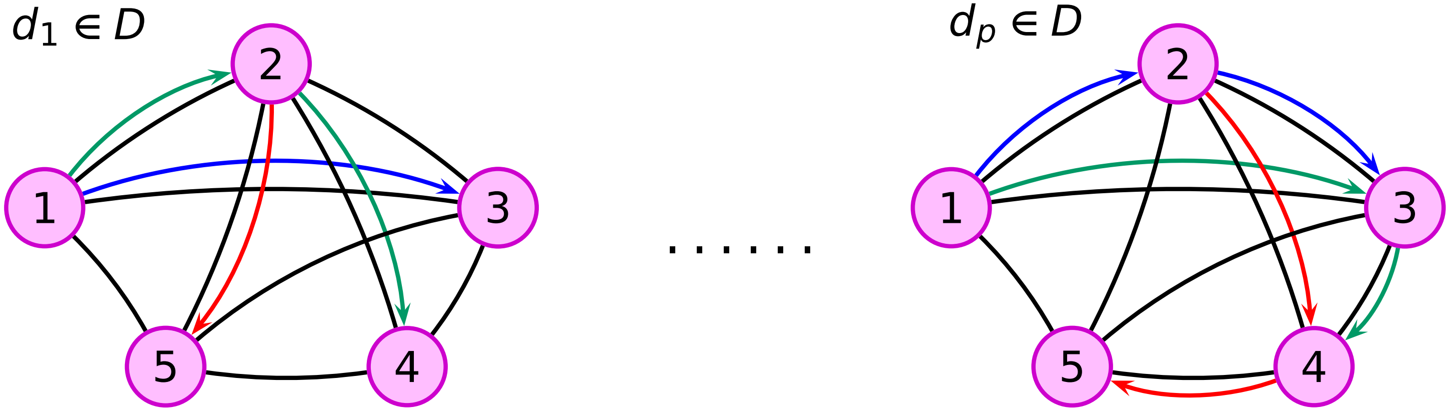
flows

- unsplittable: each commodity must be routed on a single path
- splittable: the flow for a commodity can be splitted along several paths



routing scheme

- static: the routing must be the same for all $d \in D$
- dynamic: we can choose a (possibly) different routing for every matrix



this talk

exact approach for RNL with splittable flows and dynamic routing

related problems

- Robust Network Design Problem (RND), RNL where capacities can be fractional
- Virtual Private Network Problem (VPN), RND with unsplittable flows and static routing

this talk

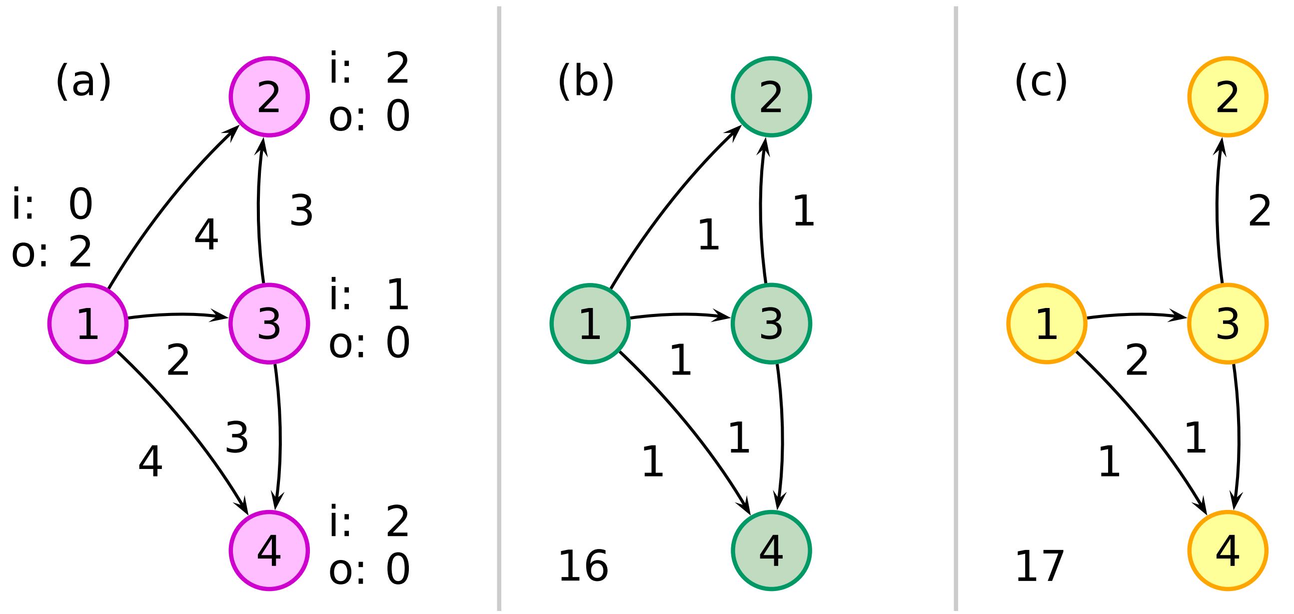
exact approach for RNL with splittable flows and dynamic routing

previous works

- many approximation results for RND and VPN [4][5][6]
- branch-and-cut-and-price for VPN [7]
- branch-and-cut for RNL with static routing [8]
- no previous exact approaches for RNL with dynamic routing

[4] Goyal, Olver, Shepherd (2009)
[5] Chekuri, Oriolo, Scutellà, Shepherd (2007)
[6] Eisenbrand, Grandoni, Oriolo, Skutella (2005)
[7] Altın, Amaldi, Belotti, Pınar (2007)
[8] Altın, Yaman, Pınar (2009)

static vs dynamic



for RND the gap between the optimal dynamic solution and the optimal static solution is $O(\log n)$ [4].

[4] Goyal, Olver, Shepherd (2009)

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{j \in N(i)} (f_{ij}^{kd} - f_{ji}^{kd}) = -d_{ki} \qquad i \in V, k \in K, d \in D \qquad (1)$$

$$\max_{d \in D} \left\{ \sum_{k \in K} (f_{ij}^{kd} + f_{ji}^{kd}) \right\} \leq x_e \qquad e = (i,j) \in E \qquad (2)$$

$$f \geq 0$$

$$x_e \in \mathbb{Z}_+$$

notation

- x_e, capacity installed on $e \in E$
- f_{ij}^{kd} flow for commodity k and demand d on $e = (i,j)$

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{j \in N(i)} (f_{ij}^{kd} - f_{ji}^{kd}) = -d_{ki} \qquad i \in V, k \in K, d \in D \qquad (1)$$

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remark

it is non-compact, while there exists a compact formulation for static routing [7][8]

[7] Altın, Amaldi, Belotti, Pınar (2007)
[8] Altın, Yaman, Pınar (2009)

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{j \in N(i)} (f_{ij}^{kd} - f_{ji}^{kd}) = -d_{ki} \qquad i \in V, k \in K, d \in D \qquad (1)$$

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$$f \geq 0$$

$$x_e \in \mathbb{Z}_+$$

question

is there a compact formulation for RNL with dynamic routing?

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{e \in E} \mu_e x_e \geq \max_{d \in D} \left\{ \sum_{k \in K} \sum_{i \in V} \ell^{\mu}_{ki} d_{ki} \right\} \qquad \mu \geq 0 \qquad (3)$$

$$x_e \in \mathbb{Z}_+$$

remarks

- due to metric inequalities [9], it is non-compact (even for NL)
- there is a non-metric capacity formulation for RNL with static routing [8]

[8] Altın, Yaman, Pınar (2009)
[9] Onaga, Kakusho (1971), Iri (1971)

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{e \in E} \mu_e x_e \geq \max_{d \in D} \left\{ \sum_{k \in K} \sum_{i \in V} \ell^{\mu}_{ki} d_{ki} \right\} \qquad \mu \geq 0 \qquad (3)$$

$$x_e \in \mathbb{Z}_+$$

question

is there a non-metric formulation for RNL with dynamic routing?

polyhedral properties

- all properties that are valid for NL(G,d) are also valid for RNL(G,D)
- all facet-defining inequalities are tight metrics

$$\sum_{e \in E} \mu_e x_e \geq R_\mu \qquad \mu \in Met_G$$

$$R_\mu = \min \left\{ \mu^T y : y \in RNL(G,D) \right\}$$

the algorithm

the formulation is non-compact, we have to use branch-and-cut

- separating tight metric inequalites is NP-hard
- no algorithm, not even heuristic, is known

separation strategy

as a first step, look for violated metric inequalites

separation of metric inequalites

given $\bar{x}$, find either an inequality

$$\sum_{e \in E} \mu_e x_e \geq \max_{d \in D} \left\{ \sum_{k \in K} \sum_{i \in V} \ell^{\mu}_{ki} d_{ki} \right\} \qquad \mu \in Met_G$$

violated by $\bar{x}$, or conclude that none exists

remarks

- for NL separating metric inequalites is easy, the separation problem can be formulated as an LP
- for RNL separating metric inequalites is difficult
- how difficult?

separation problem for metric inequalities

it can be formulated as bilevel programming problem. For other bilevel separation problems see [10]

$$\min \sum_{e\in E} \bar{x}_e \mu_e - \beta$$

$$\ell^{\mu}_{kj} \leq \ell^{\mu}_{ki} + \mu_e \quad k \in K, e \in E$$

$$\sum_{e\in E} \mu_e = 1$$

$$\mu \geq 0,\ \ell \text{ free}$$

$$\beta = \max \sum_{k\in K} \sum_{i\in V} \ell^{\mu}_{ki} d_{ki}$$

$$(\varphi_i) \sum_{k\in K} d_{ki} + \sum_{t\in V} d_{it} \leq b_i \quad i \in V$$

$$d \geq 0, d_{ii} = 0$$

$$\min \sum_{e\in E} \bar{x}_e \mu_e - \sum_{k\in K} \sum_{i\in V} \ell^{\mu}_{ki} d_{ki}$$

$$\ell^{\mu}_{kj} \leq \ell^{\mu}_{ki} + \mu_e \quad k \in K, e \in E$$

$$\sum_{e\in E} \mu_e = 1$$

$$\sum_{k\in K} d_{ki} + \sum_{t\in V} d_{it} \leq b_i \quad i \in V$$

$$\mu, d \geq 0, d_{ii} = 0\ \ell \text{ free}$$

[10] Lodi, Ralphs (2009)

separation problem for metric inequalities

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$$\ell^{\mu}_{kj} \le \ell^{\mu}_{ki} + \mu_e \quad k \in K, e \in E$$
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$$\beta = \max \sum_{k\in K}\sum_{i\in V} \ell^{\mu}_{ki} d_{ki}$$
$$(\varphi_i) \sum_{k\in K} d_{ki} + \sum_{t\in V} d_{it} \le b_i \quad i \in V$$
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$$\sum_{e\in E} \mu_e = 1$$
$$\mu \ge 0,\ \ell \text{ free}$$
$$\beta = \min \sum_{i\in V} \varphi_i b_i$$
$$(d_{ki})\ \varphi_i + \varphi_k \ge \ell^{\mu}_{ki} \quad k \in K, i \in V$$
$$\varphi \ge 0$$

[10] Lodi, Ralphs (2009)

separation problem for metric inequalities

$$\min \sum_{e \in E} \bar{x}_e \mu_e - \sum_{i \in V} \varphi_i b_i$$

$$\ell^{\mu}_{kj} \leq \ell^{\mu}_{ki} + \mu_e \qquad k \in K, e = (i,j) \in E$$

$$\sum_{e \in E} \mu_e = 1$$

$$\sum_{k \in K} d_{ki} + \sum_{t \in V} d_{it} \leq b_i \qquad i \in V$$

$$\varphi_i + \varphi_k \geq \ell^{\mu}_{ki} \qquad k \in K, i \in V$$

(compl. cond.)

$$\mu, \varphi, d \geq 0, d_{ii} = 0, \ \ell \text{ free}$$

complementarity conditions

- $d_{ij} > 0 \Rightarrow$ dual constraint satisfied with equality
- $\varphi_i > 0 \Rightarrow$ primal constraint satisfied with equality

separation problem for metric inequalities

MILP problem with binary variables and big-M constraints

question

does an easier (LP) separation problem exist?

- if so, a compact flow formulation exists
- finding a violated metric inequality means checking if $\bar{x}$ is feasible for RND
  - RND is coNP-hard for undirected and directed graphs, with asymmetric demands [5],[11]
  - symmetric hose?

[5] Chekuri, Oriolo, Scutellà, Shepherd (2007)
[11] Gupta, Kleinberg, Kumar, Rastogi, Yener (2001)

separation problem for metric inequalities

MILP problem with binary variables and big-M constraints

question

does a better (MIP) separation problem exist?

separation of {0,1} rounded metric inequalites

given $\bar{x}$, find either an inequality

$$\sum_{e \in E} \mu_e x_e \geq \max_{d \in D} \left\{ \left\lceil \sum_{k \in K} \sum_{i \in V} \ell_{ki}^{\mu} d_{ki} \right\rceil \right\} \quad \mu \in Met_G, \mu_e, \ell_{ki}^{\mu} \in \{0, 1\}$$

violated by $\bar{x}$, or conclude that none exists

remark

the separation problem is a MILP without big-M constraints

basic idea

if $\bar{x}$ is feasible for RND, then $\lceil \bar{x} \rceil$ is feasible for RNL

algorithm

- pick the current fractional solution $\bar{x}$
- round each component to the upper nearest integer obtaining an integer vector
- check feasibility

initial feasible solution

find a tree solution (static routing and unsplittable flows)

problem	|V|	|E|	opt	cuts	sec.	nodes
atlanta-1	15	22	5526	50	320.2	0
atlanta-7	15	22	5197	46	2732	2
atlanta-12	15	22	6205	60	2464.4	11
atlanta-17	15	22	3769	54	215.7	3
dfn-bwin-1	10	45	577	63	7.3	20
dfn-bwin-7	10	45	732	82	333.1	90
dfn-bwin-12	10	45	1058	65	291.5	37
dfn-bwin-17	10	45	896	34	7.3	5
dfn-gwin-1	11	47	211	32	283.2	11
dfn-gwin-7	11	47	281	72	1423.1	3
dfn-gwin-12	11	47	148	18	82.1	4
dfn-gwin-17	11	47	255	62	605.9	12
di-yuan-1	11	42	1367	30	50.9	3
di-yuan-7	11	42	1351	28	15.8	0
di-yuan-12	11	42	1327	23	10.2	0
di-yuan-17	11	42	1603	35	75.6	3
nobel-us-1	14	21	5395	64	435.7	3
nobel-us-7	14	21	6113	86	4458.6	9
nobel-us-12	14	21	6521	55	560	5
nobel-us-17	14	21	23014	61	60.4	7
pdh-1	11	34	2348	96	283.8	10
pdh-7	11	34	2988	34	50.7	3
pdh-12	11	34	2850	27	147.4	13
pdh-17	11	34	3143	28	56.7	5
polska-1	12	18	15553	57	1333.9	7
polska-7	12	18	6007	41	5.6	2
polska-12	12	18	11718	43	31.9	3
polska-17	12	18	9060	42	54.4	6

comments

- instances from the SNDlib [12]
- undirected graphs and symmetric hose
- the routine for separating $\{0, 1\}$ rounded metrics works very well, even on large instances
- it is reasonably fast
- it produces good bounds
- the bottleneck of the algorithm is the separation of metric inequalities, which is slow

[12] Orlowski, Pióro, Tomaszewski, Wessäly (2007)

this talk

an exact branch-and-cut approach for RNL with dynamic routing and splittable flows

future work

- improve the separation of metric inequalities
- symmetric hose?
  - does an LP separation problem exist?
  - does a compact flow formulation exist?
- tight metric inequalities
  - can we find inequalites stronger than rounded metrics?
  - how to formulate this problem?

thanks
for your attention